

Let  $A$  be a  $9 \times 9$  matrix with 3 distinct eigenvalues.

$\lambda_1 = 5$  has only one linearly independent eigenvector  $\vec{v}_1$ .

$\lambda_2 = 12$  has two linearly independent eigenvectors  $\vec{w}_1$  and  $\vec{w}_2$ .

$\lambda_3 = 27$  and no information about its eigenvectors is given.

1) What is the eigenspace corresponding to  $\lambda_1$ ?

2) What are possible values for the multiplicity of  $\lambda_2$ ?

3) How many linearly independent eigenvectors can there be for  $\lambda_3$ ?

4) What are possible values for  $|A|$ ?

5) What is the leading term in the characteristic polynomial of  $A$ ? That is, when written in the standard order, what is the first term in  $A - xI_9$ ?